An open quantum system approach to EPR correlations in $K^0\bar{K}^0$ system

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Abstract

We find the time evolution of the system of two non-interacting unstable particles, distinguishable as well as identical ones, in arbitrary reference frame having only the Kraus operators governing the evolution of its components in the rest frame. We than calculate in the rigorous way Einstein–Podolsky–Rosen quantum correlation functions for $K^0\bar{K}^0$ system in the singlet state taking into account CP-violation and decoherence and show that the results are exactly the same despite the fact we treat kaons as distinguishable or identical particles which means that the statistics of the particles plays no role, at least in considered cases.

1 Introduction

In the recent years the possibility of testing Bell-CHSH inequalities [1,2] in the system of correlated neutral mesons has attracted some attention (see e.g. [3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19]), because kaons and B-mesons are detected with higher efficiency than photons which maybe allows one to close the detection loophole (see e.g. [5]).

The crucial point in studying quantum correlations in the system of unstable particles, say $K^0\bar{K}^0$, is the choice of a model describing time evolution of the system under consideration. It is obvious that this choice depends on what

physical aspects we want to neglect and what we want to focus on. For example, if we want to focus our attention on decay of particles governed by the Geiger–Nutall law neglecting the evolution of decay products and other physical processes like e.g. decoherence, we can use the standard Weisskopf–Wigner approach [20,21]. The price we must pay for this choice are some ambiguities, due to the non-Hermitian Hamiltonian of Weisskopf–Wigner model, when we want to consider a series of consecutive measurements. However, if we mainly want to take into accout decoherence we can do our calculations in the framework of many models considered in the literature (see e.g. [22,23,24,25]). The common feature of all these models is taking into account only the two-particle sector of density matrix describing the state of the $K^0\bar{K}^0$ system, neglecting the one-particle and zero-particle sectors which arise during the time evolution due to the decay process.

Recently, we have introduced a model [26] of time evolution of a neutral kaon in its rest frame, which allows one to describe the $K^0\bar{K}^0$ system in more realistic way. In the framework of our model, based on the theory of open quantum systems (see e.g. [27,28]), we can take into account that the state of the system evolves from a two-particle state to the zero-particle one through states being a mixture of all possible states. This model is based on the following assumptions. (i) A neutral kaon is treated as an open quantum system, and the decay products are regarded as a part of the environment. (ii) The system can be found either in a particle state, or in the state of absence of the particle, which we call (in analogy to the quantum field theory) the vacuum state, and denote by $|0\rangle$. Therefore the Hilbert space of the neutral kaon–vacuum system is a direct sum of one-particle Hilbert space and one-dimensional space $\mathcal{H}_0$ spanned by $|0\rangle$. (iii) The superselection rule prohibiting the superposition of the particle and vacuum holds. (iv) The time evolution of the system, consistent with the Geiger–Nutall law and allowing for CP symmetry violation, is given by a family of completely positive trace preserving maps forming a one-parameter dynamical semigroup. Complete positivity implies that the time evolution of a state of the system can be written in the operator-sum (Kraus) representation [29] (which immediately allows one to find the time evolution of non-interacting particles)

$$\hat{\varrho}(t) = \sum_i \hat{E}_i(t)\hat{\varrho}(0)\hat{E}_i^\dagger(t),\tag{1}$$

and the trace preservation requirement leads to the condition

$$\sum_i \hat{E}_i^\dagger(t)\hat{E}_i(t) = \mathbf{1}\,.\tag{2}$$

As a remarkable side effect of complete positivity of time evolution of the system we obtained the upper bound for the decoherence parameter (see [26]). Here we would like to point out that we used the dynamical semigroup approach for the entire evolution of the system, not only for the description of

its decoherence, as was done in [22,23,24,25].

The paper is organized as follows. In Section 2 we generalize our description of time evolution of an unstable particle in its rest frame to the case of an arbitrary reference frame. Then, in Section 3, we show how the evolution of the system of two unstable non-interacting particles can be found when we know the Kraus operators governing the evolution of its components, both for distinguishable and identical particles case. Finally, in Section 4 we apply these results in calculation of some quantum correlation functions for $K^0 \bar{K}^0$ system in the singlet state. And we show that the obtained results are exactly the same for kaons treated as distinguishable or identical particles.

2 Time evolution of an unstable particle in an arbitrary reference frame

Let us assume that we know the Kraus representation of time evolution of an unstable particle in its rest frame in which the four-momentum is $\tilde{k} = (m, \vec{0})$. Let us answer the question, how this evolution looks like from the point of view of an observer in the laboratory frame, in which the particle has four-momentum $k = (k^0, \vec{k})$. Let L_k be the Lorentz boost from the rest frame to the laboratory frame, i.e. $k = L_k \tilde{k}$. When we denote the inner degrees of freedom (e.g. strangeness, bottom) as s, than the state of particle with four-momentum k can be denoted as $|s, k\rangle$ (in this notation, $|0\rangle \equiv |0, 0\rangle$). Since we deal only with (pseudo) scalar particles (e.g. kaons), $|s, k\rangle = U(L_k)|s, \tilde{k}\rangle$, which implies that the density operator in laboratory frame has the following form

$$\hat{\varrho}^{\text{lab}}(t) = U(L_k)\hat{\varrho}(\tau^k)U^\dagger(L_k)$$
$$= \sum_i \hat{E}_i^{k\,\text{lab}}(\tau^k)\hat{\varrho}^{\text{lab}}(0)(\hat{E}_i^{k\,\text{lab}}(\tau^k))^\dagger, \tag{3}$$

where the proper time in the rest frame of the particle is denoted by τ^k because it is convenient to express it by means of k and the laboratory time t:

$$\tau^k = \frac{t}{\sqrt{1 + \frac{\vec{k}^2}{m^2}}} \equiv \frac{t}{\gamma^k}, \tag{4}$$

and $\hat{E}_i^{k\,\text{lab}}(\tau^k) = U(L_k)\hat{E}_i(\tau^k)U^\dagger(L_k)$. By means of the decomposition $\hat{E}_i(\tau^k) = \sum_{s,s'} E_i^{ss'}(\tau^k)|s, \tilde{k}\rangle\langle s', \tilde{k}|$ (see Appendix A), we get

$$\hat{E}_i^{k\,\text{lab}}(\tau^k) = \sum_{s,s'} E_i^{ss'}(\tau^k)|s, k\rangle\langle s', k|. \tag{5}$$

It means that the matrix elements of $\hat{E}_i^{k\,\text{lab}}(\tau^k)$ in the basis $\{|s, k\rangle\}$ are exactly the same as the matrix elements of $\hat{E}_i(\tau^k)$ in the basis $\{|s, \tilde{k}\rangle\}$. Note that this

holds only for (pseudo) scalar particles.

Taking into account the form of the Kraus operators given in Appendix A, the evolution (3) can be easily extended to the case in which the particle state is the superposition (or mixture) of a few different momentum eigenstates. In this case the space of states of the system is spanned by the orthogonal vectors $|s, k\rangle$ and vacuum, but now the four-momentum k can take an arbitrary but finite number of different values. We denote the set of admissible four-momenta by Q (for example, in the case of two identical particles discussed in the next section, Q consists of only two elements $Q = \{p, q\}$). The time evolution of the density operator describing such a state has the following form

$$
\begin{aligned}
\hat{\varrho}^{\text{lab}}(t) &= \hat{E}_0 \hat{\varrho}^{\text{lab}}(0) \hat{E}_0^\dagger \\
&\quad + \sum_{k \in Q} \sum_{i=1}^{5} \hat{E}_i^{k\,\text{lab}}(\tau^k) \hat{\varrho}^{\text{lab}}(0) (\hat{E}_i^{k\,\text{lab}}(\tau^k))^\dagger,
\end{aligned}
\tag{6}
$$

where the Kraus operators $\hat{E}_0 = \hat{E}_0^{\text{lab}} = |0,0\rangle\langle 0,0|$ and $\hat{E}_i^{k\,\text{lab}}(\tau^k)$, are given in Appendix A. Note that when Q consists of only one element Eq. (6) reduces to (3).

3 Two unstable particles as open quantum system

Let us consider two unstable particles $\mathcal{A}$ and $\mathcal{B}$ (this approach can be easily extended to the multiparticle case), with four-momenta p and q in the laboratory frame, respectively. The space of states of the whole system is

$$
\begin{aligned}
\mathcal{H} &= (\mathcal{H}_\mathcal{A} \oplus \mathcal{H}_0) \otimes (\mathcal{H}_\mathcal{B} \oplus \mathcal{H}_0) \\
&= (\mathcal{H}_\mathcal{A} \otimes \mathcal{H}_\mathcal{B}) \oplus (\mathcal{H}_\mathcal{A} \otimes \mathcal{H}_0 \oplus \mathcal{H}_0 \otimes \mathcal{H}_\mathcal{B}) \oplus (\mathcal{H}_0 \otimes \mathcal{H}_0),
\end{aligned}
\tag{7}
$$

so one can easily see that the system can be either in two-particle state $(\mathcal{H}_\mathcal{A} \otimes \mathcal{H}_\mathcal{B})$, or in one-particle state $(\mathcal{H}_\mathcal{A} \otimes \mathcal{H}_0 \oplus \mathcal{H}_0 \otimes \mathcal{H}_\mathcal{B})$, or in the vacuum state $(\mathcal{H}_0 \otimes \mathcal{H}_0)$. Such a system is an open one, so its evolution is represented by a completely positive map and can be written in the operator-sum representation

$$
\hat{\varrho}_{\mathcal{A}\mathcal{B}}^{\text{lab}}(t) = \sum_{k \in Q, k' \in Q'} \sum_{i,j} \hat{E}_{ij}^{kk'}(t) \hat{\varrho}_{\mathcal{A}\mathcal{B}}^{\text{lab}}(0) (\hat{E}_{ij}^{kk'}(t))^\dagger,
\tag{8}
$$

where, under a reasonable assumption that the particles do not interact, we define

$$
\hat{E}_{ij}^{kk'}(t) = \hat{E}_i^{k\,\text{lab}}(\tau^k) \otimes \hat{E}_j^{k'\,\text{lab}}(\tau^{k'}).
\tag{9}
$$

$\hat{E}_i^{k\,\text{lab}}(\tau)$ are the Kraus operators governing the time evolution of a one-particle system, and $i, j = 0, \ldots, 5$ (see Appendix A), and $\hat{E}_0^{k\,\text{lab}}(\tau) = \hat{E}_0$. The normalization condition $\sum_{k \in Q, k' \in Q'} \sum_{i,j} (\hat{E}_{ij}^{kk'}(t))^\dagger \hat{E}_{ij}^{kk'}(t) = \mathbf{1} \otimes \mathbf{1}$ is fulfilled, which

4

follows from corresponding normalization conditions for one-particle Kraus operators (A.2). It is clear that for *distinguishable* particles we have $Q = \{p\}$ and $Q' = \{q\}$. However in the case of *identical* particles one does not know which of them carries which four-momentum or flavour, thus the space of the whole system must be symmetrized, and $Q = Q' = \{p, q\}$. Let us introduce the permutation operator $\mathcal{P}$ such that $\mathcal{P}(|s, k\rangle \otimes |s', k'\rangle) = |s', k'\rangle \otimes |s, k\rangle$; then

$$\mathcal{P}\hat{\varrho}^{\text{lab}}_{\mathcal{AB}}(t)\mathcal{P} = \hat{\varrho}^{\text{lab}}_{\mathcal{AB}}(t). \tag{10}$$

Moreover any observable $\hat{O}$ must also preserve symmetrization, i.e.:

$$\mathcal{P} \, \hat{O} \, \mathcal{P} = \hat{O}. \tag{11}$$

One can easily verify that if $\hat{\varrho}^{\text{lab}}_{\mathcal{AB}}(0)$ fulfils the condition (10), than also $\hat{\varrho}^{\text{lab}}_{\mathcal{AB}}(t)$ obtained from (8) fulfils it, so the time evolution of identical particles is governed by the same evolution law (8) as the time evolution of the distinguishable ones.

In EPR-type experiments it is of great importance that the measurements performed by distant observers, say Alice and Bob, must be local. The locality means that the observables are restricted to some specific region usually interpreted as the region of detector (see e.g. [30]). It can be achieved experimentally by assuming, for example, that Alice's detector can register only particles with four-momentum p and Bob's only those with four-momentum q (directions of momenta $\vec{p}$ and $\vec{q}$ must be sufficiently different)[1]. Thus, only the particle carrying four-momentum p and q, respectively can reach Alice's and Bob's detectors.

The space of states of a two-particle system is the tensor product $(\mathcal{H}_{\mathcal{A}} \oplus \mathcal{H}_0) \otimes (\mathcal{H}_{\mathcal{B}} \oplus \mathcal{H}_0)$ (see (7)). In the case of distinguishable particles $\mathcal{H}_{\mathcal{A}}$ is spanned by $\{|s, p\rangle\}$ and $\mathcal{H}_{\mathcal{B}}$ by $\{|s', q\rangle\}$, so Alice's detector registers only the particles with states from subspace $\mathcal{H}_{\mathcal{A}} \oplus \mathcal{H}_0$ and Bob's only those from $\mathcal{H}_{\mathcal{B}} \oplus \mathcal{H}_0$. On the other hand, in the case of indistinguishable particles $\mathcal{H}_{\mathcal{A}}$ and $\mathcal{H}_{\mathcal{B}}$ must be identical and both spanned by the same set of vectors $\{|s, p\rangle, |s', q\rangle\}$. The physical Hilbert space is symmetric subspace of $\mathcal{H}$, i.e. $\frac{1}{2}(\mathbf{1} + \mathcal{P})\mathcal{H}$, therefore one cannot associate specific one-particle Hilbert space with a given observer.

4 Quantum correlations in the neutral kaon system

Let us consider two neutral kaons in a given initial state $\hat{\varrho}^{\text{lab}}_{\mathcal{AB}}(0)$, and two distant observers, Alice and Bob, in the same laboratory frame. Alice can

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[1] Actually the sharp momentum states cannot be achieved because of the uncertainty principle and finite volume of detector.

measure the flavour of kaons only with four-momentum p and Bob only those with four-momentum q, therefore their observables commute, i.e.:

$$[\hat{A}^p, \hat{B}^q] = 0. \tag{12}$$

Hereafter, we omit the superscript 'lab', because in the sequel we consider only density operators as seen from the laboratory frame.

4.1 Distinguishable case

In this subsection we treat the kaons as distinguishable particles, as it is usually done to simplify calculations. The Hilbert space of the first kaon is spanned by set of orthonormal vectors $\left\{|K^0, p\rangle, |\bar{K}^0, p\rangle, |0, 0\rangle\right\}$ and the Hilbert space of the second one by $\left\{|K^0, q\rangle, |\bar{K}^0, q\rangle, |0, 0\rangle\right\}$.

Suppose that at time t_A Alice measures an observable $\hat{A}^p = A \otimes \mathbf{1}$ with the spectral decomposition $\sum_a a \left(\Pi_a \otimes \mathbf{1}\right)$. The density operator just before the measurement is

$$\hat{\varrho}_{\mathcal{AB}}(t_A) = \sum_{ij} \hat{E}_{ij}^{pq}(t_A) \hat{\varrho}_{\mathcal{AB}}(0)(\hat{E}_{ij}^{pq}(t_A))^\dagger, \tag{13}$$

(cf. (8)), and when the outcome of the measurement is a, the state reduces to

$$\hat{\varrho}_{\mathcal{AB}}^a(t_A) = \frac{(\Pi_a \otimes \mathbf{1})\hat{\varrho}_{\mathcal{AB}}(t_A)(\Pi_a \otimes \mathbf{1})}{p_a(t_A)}, \tag{14}$$

where $p_a(t_A)$ is the probability of measuring a at time t_A. Next, at time t_B, Bob performs the measurement of $\hat{B}^q = \mathbf{1} \otimes B$ with spectral decomposition $\sum_b b \left(\mathbf{1} \otimes \Pi_b\right)$. Just before his measurement the state is

$$\hat{\varrho}_{\mathcal{AB}}^a(t_B) = \sum_{ij} \hat{E}_{ij}^{pq}(t_B - t_A)\hat{\varrho}_{\mathcal{AB}}^a(t_A)(\hat{E}_{ij}^{pq}(t_B - t_A))^\dagger. \tag{15}$$

The conditional probability that Bob's outcome is b provided that Alice's was a is

$$p_{b|a}(t_B) = \text{Tr}\left\{(\mathbf{1} \otimes \Pi_b)\, \hat{\varrho}_{\mathcal{AB}}^a(t_B)\, (\mathbf{1} \otimes \Pi_b)\right\}. \tag{16}$$

By means of (13)–(16), the joint probability

$$p_{ab}(t_A, t_B) = p_a(t_A)p_{b|a}(t_B), \tag{17}$$

that Alice's and Bob's outcomes are a and b, respectively is given by the formula

$$p_{ab}(t_A, t_B) = \text{Tr}\left\{\hat{\varrho}_{\mathcal{AB}}(0)\left[\sum_i (\hat{E}_i^p\left(\tau_A^p\right))^\dagger \Pi_a \hat{E}_i^p\left(\tau_A^p\right) \otimes \sum_j (\hat{E}_j^q\left(\tau_B^q\right))^\dagger \Pi_b \hat{E}_j^q\left(\tau_B^q\right)\right]\right\}. \tag{18}$$

6

Then the correlation function between the outcomes

$$C_{A^pB^q}(t_A, t_B) = \sum_{ab} ab\, p_{ab}(t_A, t_B), \qquad (19)$$

takes the form

$$C_{A^pB^q}(t_A, t_B) = \text{Tr}\left\{\hat{\varrho}_{\mathcal{AB}}(0)\left[A(\tau_A^p) \otimes B(\tau_B^q)\right]\right\}, \qquad (20)$$

where $A(\tau_A^p) = \sum_i(\hat{E}_i^p\,(\tau_A^p))^\dagger A\hat{E}_i^p\,(\tau_A^p)$, and $B(\tau_B^q) = \sum_j(\hat{E}_j^q\,(\tau_B^q))^\dagger B\hat{E}_j^q\,(\tau_B^q)$.

Now we calculate explicitly a few correlation functions and probabilities in the system of neutral kaons in the pure entangled state $J^{PC} = 1^{--}$ (produced through the reaction $e^+e^- \to \phi(1020) \to K^0\bar{K}^0$)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|K^0, p\rangle \otimes |\bar{K}^0, q\rangle - |\bar{K}^0, p\rangle \otimes |K^0, q\rangle\right) \qquad (21)$$

to show how our model works. We do not neglect CP violation and decoherence, despite the fact that CPLEAR experiment at CERN (where $K^0\bar{K}^0$ pairs are produced in the $p\bar{p}$ collider) is not sensitive to CP violating effects. Let us begin with the strangeness correlations. The strangeness operator takes the form

$$S^k = |K^0, k\rangle\langle K^0, k| - |\bar{K}^0, k\rangle\langle\bar{K}^0, k|, \qquad (22)$$

where $k = p, q$, and $\hat{A}^p = S^p \otimes \mathbf{1}$, $\hat{B}^q = \mathbf{1} \otimes S^q$. It is easy to show that the corresponding correlation function has the following form

$$C_{S^pS^q}(t_A, t_B) = -\frac{1}{1-\delta_L^2}\left[\mathrm{e}^{-(\Gamma+\lambda)(\tau_A^p+\tau_B^q)}\cos(\Delta m\Delta\tau)\right.$$
$$\left. -\tfrac{1}{2}\delta_L^2\left(\mathrm{e}^{-\Gamma_S\tau_A^p-\Gamma_L\tau_B^q} + \mathrm{e}^{-\Gamma_L\tau_A^p-\Gamma_S\tau_B^q}\right)\right], \quad (23)$$

where τ_A^p and τ_B^q stand for the proper times, $\Delta\tau = \tau_B^q - \tau_A^p$, $\delta_L = 2\Re(\epsilon)/(1+|\epsilon|^2)$ (ϵ is a small complex CP-violation parameter), $\Gamma = \tfrac{1}{2}(\Gamma_S+\Gamma_L)$ (Γ_S and Γ_L are the decay widths of short and long living states of neutral kaon, respectively), $\Delta m = m_L - m_S$ (m_S and m_L are masses of short and long living states of neutral kaon, respectively) and λ is a decoherence parameter, representing interaction between one-particle system and the environment. On the other hand, the strangeness operators defined above have three different eigenvalues ± 1 and 0, but the observables considered in Bell-CHSH inequalities have only two different eigenvalues ± 1. Such a dichotomic observable is, for example, an observable answering the question whether one registers a kaon (anti-kaon) (then the result of the measurement is $+1$), or not (then the result is -1). Such a case was investigated in [3]. Denoting this observable as D_+ (D_-), in one-particle case we have

$$D_\pm^k = \pm|K^0, k\rangle\langle K^0, k| \mp |\bar{K}^0, k\rangle\langle\bar{K}^0, k| - |0, 0\rangle\langle 0, 0|. \qquad (24)$$

It is easy to show that in the case when $\hat{A}^p = D_+^p \otimes \mathbf{1}$ and $\hat{B}^q = \mathbf{1} \otimes D_+^q$, we get

$$
\begin{aligned}
C_{D_+^p D_+^q}(t_A, t_B) = {}& 1 - \frac{1+\delta_L}{1-\delta_L}\Big[e^{-(\Gamma+\lambda)(\tau_A^p + \tau_B^q)} \cos(\Delta m \Delta \tau) \\
& - \tfrac{1}{2}\left(e^{-\Gamma_S \tau_A^p - \Gamma_L \tau_B^q} + e^{-\Gamma_L \tau_A^p - \Gamma_S \tau_B^q} \right) \Big] \\
& - \frac{1}{2(1-\delta_L)}\left(e^{-\Gamma_S \tau_A^p} + e^{-\Gamma_S \tau_B^q} + e^{-\Gamma_L \tau_A^p} + e^{-\Gamma_L \tau_B^q} \right) \\
& + \frac{\delta_L}{1-\delta_L}\left(e^{-(\Gamma+\lambda)\tau_A^p}\cos(\Delta m \tau_A^p) + e^{-(\Gamma+\lambda)\tau_B^q}\cos(\Delta m \tau_B^q) \right), \quad (25)
\end{aligned}
$$

and when $\hat{A}^p = D_+^p \otimes \mathbf{1}$ and $\hat{B}^q = \mathbf{1} \otimes D_-^q$, we have

$$
\begin{aligned}
C_{D_+^p D_-^q}(t_A, t_B) = {}& 1 + e^{-(\Gamma+\lambda)(\tau_A^p + \tau_B^q)}\cos(\Delta m \Delta \tau) \\
& - \frac{1}{2(1-\delta_L)}\left(e^{-\tau_A^p \Gamma_S} + e^{-\tau_A^p \Gamma_L} - 2\delta_L\, e^{-\tau_A^p (\Gamma+\lambda)}\cos(\Delta m \tau_A^p) \right) \\
& - \frac{1}{2(1+\delta_L)}\left(e^{-\tau_B^q \Gamma_S} + e^{-\tau_B^q \Gamma_L} + 2\delta_L\, e^{-\tau_B^q(\Gamma+\lambda)}\cos(\Delta m \tau_B^q) \right) \\
& + \tfrac{1}{2}\left(e^{-\tau_A^p \Gamma_S - \tau_B^q \Gamma_L} + e^{-\tau_A^p \Gamma_L - \tau_B^q \Gamma_S} \right). \quad (26)
\end{aligned}
$$

Of course, we could find the above quantum correlation functions directly from the definition (19) finding appropriate probabilities. For example, one can easily find the stangeness correlation function $C_{S^p S^q}(t_A, t_B)$ knowing that
(i) the probability that Alice's detector registers K^0 at t_A and Bob's K^0 at t_B is

$$
\begin{aligned}
p_{K^0, K^0}(t_A, t_B) = {}& \frac{1}{8}\frac{1+\delta_L}{1-\delta_L}\Big[e^{-\Gamma_S \tau_A^p - \Gamma_L \tau_B^q} + e^{-\Gamma_S \tau_B^q - \Gamma_L \tau_A^p} \\
& - 2\, e^{-(\Gamma+\lambda)(\tau_A^p + \tau_B^q)}\cos(\Delta m \Delta \tau) \Big], \quad (27\text{a})
\end{aligned}
$$

(ii) the probability that Alice's detector registers $\bar{K}^0$ at t_A and Bob's $\bar{K}^0$ at t_B is

$$
\begin{aligned}
p_{\bar{K}^0, \bar{K}^0}(t_A, t_B) = {}& \frac{1}{8}\frac{1-\delta_L}{1+\delta_L}\Big[e^{-\Gamma_S \tau_A^p - \Gamma_L \tau_B^q} + e^{-\Gamma_S \tau_B^q - \Gamma_L \tau_A^p} \\
& - 2\, e^{-(\Gamma+\lambda)(\tau_A^p + \tau_B^q)}\cos(\Delta m \Delta \tau) \Big], \quad (27\text{b})
\end{aligned}
$$

(iii) the probabilities that Alice's detector registers $\bar{K}^0$ at t_A and Bob's K^0 at t_B, and that Alice's detector registers K^0 at t_A and Bob's $\bar{K}^0$ at t_B are

$$
\begin{aligned}
p_{\bar{K}^0, K^0}(t_A, t_B) = {}& p_{K^0, \bar{K}^0}(t_A, t_B) \\
= {}& \frac{1}{8}\Big[e^{-\Gamma_S \tau_A^p - \Gamma_L \tau_B^q} + e^{-\Gamma_S \tau_B^q - \Gamma_L \tau_A^p} + 2\, e^{-(\Gamma+\lambda)(\tau_A^p + \tau_B^q)}\cos(\Delta m \Delta \tau) \Big]. \\
& \hspace{10cm} (27\text{c})
\end{aligned}
$$

In [3] the correlation function $C_{D_+^p D_+^q}(t_A, t_B)$ and probabilities $p_{K^0, K^0}(t_A, t_B)$, $p_{\bar{K}^0, \bar{K}^0}(t_A, t_B)$ and $p_{\bar{K}^0, K^0}(t_A, t_B)$ were found under condition $\delta_L = \lambda = 0$, i.e. without CP-violation and decoherence. Of course, when we put $\delta_L = \lambda = 0$ in (25) and (27) we arrive at Bertlmann's results, if we only take into account differences in conventions.

4.2 Indistinguishable case

Now let us consider the same situation as in the previous subsection but with more realistic assumption that kaons are indistinguishable particles. Suppose that Alice measures $\hat{A}^p$ at time t_A and Bob measures $\hat{B}^q$ at time t_B. Both of the observables must fulfil (11). After analogous calculations as in the previous subsection, using (12), we get

$$C_{\hat{A}^p \hat{B}^q}(t_A, t_B) = \mathrm{Tr}\left[\hat{\varrho}_{\mathcal{AB}}(0)\hat{A}^p(\tau_A^p)\hat{B}^q(\tau_B^q)\right], \qquad (28)$$

where

$$\hat{A}^p(\tau_A^p) = \sum_{ij}(\hat{E}_{ij}^{pq}(t_A))^\dagger \hat{A}^p \hat{E}_{ij}^{pq}(t_A) \quad \text{and} \quad \hat{B}^q(\tau_B^q) = \sum_{ij}(\hat{E}_{ij}^{pq}(t_B))^\dagger \hat{B}^q \hat{E}_{ij}^{pq}(t_B).$$

Now let us calculate the same correlation functions as before. First, we have to note that in the case of indistinguishable particles the initial state has different form

$$|\psi\rangle = \frac{1}{2}\left(|K^0, p\rangle \otimes |\bar{K}^0, q\rangle + |\bar{K}^0, q\rangle \otimes |K^0, p\rangle \right.$$
$$\left. - |\bar{K}^0, p\rangle \otimes |K^0, q\rangle - |K^0, q\rangle \otimes |\bar{K}^0, p\rangle\right). \quad (29)$$

Second, we have to construct observables that answer the same questions as the observables used in the case of distinguishable kaons. The strangeness operators $\hat{S}^k$ take the form

$$\hat{S}^k = S^k \otimes \mathbf{1} + \mathbf{1} \otimes S^k, \qquad (30)$$

where k takes the value p or q, and S^k was defined in the previous subsection. Observables $\hat{D}_\pm^k$ cannot be constructed in analogy to (30) by means of (24)

and symmetrization[2]. Now, the form of $\hat{D}^k_+$ is

$$\hat{D}^k_+ = 2\left(|K^0,k\rangle\langle K^0,k| \otimes \mathbf{1} + \mathbf{1} \otimes |K^0,k\rangle\langle K^0,k|\right)$$
$$- \mathbf{1} \otimes \mathbf{1} - |K^0,k\rangle\langle K^0,k| \otimes |K^0,k\rangle\langle K^0,k|. \quad (31)$$

It yields $+2$ when both of the particles are kaons with four-momentum k, $+1$ when one of the particles is a kaon with four-momentum k, and -1, when there is no such kaon. It is not dichotomic, but it is not a problem because in state (29) the probability of measuring two kaons with the same four-momentum equals zero. $\hat{D}^k_-$ takes the analogous form to $\hat{D}^k_+$

$$\hat{D}^k_- = 2\left(|\bar{K}^0,k\rangle\langle\bar{K}^0,k| \otimes \mathbf{1} + \mathbf{1} \otimes |\bar{K}^0,k\rangle\langle\bar{K}^0,k|\right)$$
$$- \mathbf{1} \otimes \mathbf{1} - |\bar{K}^0,k\rangle\langle\bar{K}^0,k| \otimes |\bar{K}^0,k\rangle\langle\bar{K}^0,k|. \quad (32)$$

It is easy to check that the correlation functions $C_{\hat{S}^p\hat{S}^q}(t_A,t_B)$, $C_{\hat{D}^p_+\hat{D}^q_+}(t_A,t_B)$ and $C_{\hat{D}^p_+\hat{D}^q_-}(t_A,t_B)$ are exactly the same as in the distinguishable particles case (23), (25) and (26). It is quite remarkable that for the observables considered above, it does not matter whether we treat kaons as indistinguishable particles or not, at least in the singlet state. Of course, it is easier to carry out all calculations on the assumption that kaons are distinguishable particles, as it is usually done.

5   Conclusions

We have shown that having the Kraus representation of time evolution of an unstable particle in its rest frame it is possible to find the evolution of the particle in an arbitrary reference frame. Moreover, we have also shown that taking into account the form of Kraus operators one can extend the time evolution of the system to the case in which the state of the particle is the superposition or mixture of a few different momentum eigenstates. Next, we have found the time evolution of two non-interacting unstable particles, distinguishable as well as identical ones. Finally, we have applied these results in calculation of some quantum correlation functions for $K^0\bar{K}^0$ system in the singlet state assuming CP-violation and decoherence. And it turned out that the results are exactly the same either we treat kaons as distinguishable particles or identical ones. Therefore, one can neglect the fact that kaons are identical particles and treat them as distinguishable ones, at least in the cases

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[2] When we calculate the spectral decomposition of, say, $D^k_+ \otimes \mathbf{1} + \mathbf{1} \otimes D^k_+$, where $D^k_+ = 2|K^0,k\rangle\langle K^0,k| - \mathbf{1}$, we find out that it has eigenvalues equal ± 2 and 0, so it does not answer the question, whether the particle is kaon carrying momentum k, or not.

considered in this paper. It is still an open question whether the statistics of the particles plays no role in the general case or not.

We would like to point out that all results presented in this paper will be valid also for B-mesons after appropriate change of notation, because kaons and B-mesons evolve according to the same scheme.

Acknowledgements

A Time evolution of K^0

Let us assume that the space of states of the system is spanned by the orthogonal vectors: vacuum $|0,0\rangle$ and $|s,k\rangle$, where s denotes inner degrees of freedom and four-momentum k belongs to the finite set Q of admissible four-momenta. The Kraus operators $\hat{E}_i^k(t) = \sum_{s,s'} E_i^{ss'}(\tau^k)|s,k\rangle\langle s',k|$, describing evolution (6) of the kaon which state can be a superposition (or mixture) of different momentum eigenstates, have the following form

$$\hat{E}_0^k(\tau^k) = \hat{E}_0 = |0,0\rangle\langle 0,0|, \tag{A.1a}$$

$$\begin{aligned}
\hat{E}_1^k(\tau^k) &= \frac{1}{2}\left(e^{-\tau^k(\lambda+2im_S+\Gamma_S)/2} + e^{-\tau^k(\lambda+2im_L+\Gamma_L)/2}\right) \\
&\quad \times \left(|K^0,k\rangle\langle K^0,k| + |\bar{K}^0,k\rangle\langle \bar{K}^0,k|\right) \\
&\quad + \frac{1}{2}\left(e^{-\tau^k(\lambda+2im_S+\Gamma_S)/2} - e^{-\tau^k(\lambda+2im_L+\Gamma_L)/2}\right) \\
&\quad \times \left(\frac{1+\epsilon}{1-\epsilon}|K^0,k\rangle\langle \bar{K}^0,k| + \frac{1-\epsilon}{1+\epsilon}|\bar{K}^0,k\rangle\langle K^0,k|\right),
\end{aligned} \tag{A.1b}$$

$$\begin{aligned}
\hat{E}_2^k(\tau^k) &= \sqrt{\frac{1+|\epsilon^2|}{2}}\left(1 - e^{-\tau^k\Gamma_S} - \delta_L^2\frac{\left|1 - e^{-\tau^k(\Gamma+\lambda-i\Delta m)}\right|^2}{1 - e^{-\tau^k\Gamma_L}}\right)^{\frac{1}{2}} \\
&\quad \times \left(\frac{1}{1+\epsilon}|0,0\rangle\langle K^0,k| + \frac{1}{1-\epsilon}|0,0\rangle\langle \bar{K}^0,k|\right),
\end{aligned} \tag{A.1c}$$

$$\hat{E}_3^k(\tau^k) = \sqrt{\frac{1+|\epsilon|^2}{2(1-\mathrm{e}^{-\tau^k\Gamma_L})}} \left(\frac{1-\mathrm{e}^{-\tau^k\Gamma_L}+\delta_L-\mathrm{e}^{-\tau^k(\lambda-i\Delta m+\Gamma)}\,\delta_L}{1+\epsilon}|0,0\rangle\langle K^0,k| \right.$$
$$\left. -\frac{1-\mathrm{e}^{-\tau^k\Gamma_L}-\delta_L+\mathrm{e}^{-\tau^k(\lambda-i\Delta m+\Gamma)}\,\delta_L}{1-\epsilon}|0,0\rangle\langle \bar{K}^0,k| \right), \qquad (A.1d)$$

$$\hat{E}_4^k(\tau^k) = \frac{1}{2}\,\mathrm{e}^{-\tau^k\Gamma_S/2}\sqrt{1-\mathrm{e}^{-\tau^k\lambda}} \left(|K^0,k\rangle\langle K^0,k| + |\bar{K}^0,k\rangle\langle \bar{K}^0,k| \right.$$
$$\left. +\frac{1+\epsilon}{1-\epsilon}|K^0,k\rangle\langle \bar{K}^0,k| + \frac{1-\epsilon}{1+\epsilon}|\bar{K}^0,k\rangle\langle K^0,k| \right), \qquad (A.1e)$$

$$\hat{E}_5^k(\tau^k) = \frac{1}{2}\,\mathrm{e}^{-\tau^k\Gamma_L/2}\sqrt{1-\mathrm{e}^{-\tau^k\lambda}} \left(|K^0,k\rangle\langle K^0,k| + |\bar{K}^0,k\rangle\langle \bar{K}^0,k| \right.$$
$$\left. -\frac{1+\epsilon}{1-\epsilon}|K^0,k\rangle\langle \bar{K}^0,k| - \frac{1-\epsilon}{1+\epsilon}|\bar{K}^0,k\rangle\langle K^0,k| \right), \qquad (A.1f)$$

where $\tau^k = t/\gamma^k$, ϵ is a small complex CP-violation parameter, $\delta_L = 2\Re(\epsilon)/(1+|\epsilon|^2)$, Γ_S and Γ_L are the decay widths of K_S^0 and K_L^0 (short and long living states of neutral kaon), respectively, $\Gamma = \frac{1}{2}(\Gamma_S+\Gamma_L)$, m_S and m_L are masses of K_S^0 and K_L^0, respectively, $\Delta m = m_L - m_S$, and λ is a decoherence parameter, representing interaction between one-particle system and the environment. In comparison to [26] we use different, more convenient set of Kraus operators, leading to the same evolution. It is easy to check that the normalization condition

$$\hat{E}_0^\dagger \hat{E}_0 + \sum_{k\in Q}\sum_{i=1}^{5}(\hat{E}_i^{k\,\mathrm{lab}}(\tau^k))^\dagger \hat{E}_i^{k\,\mathrm{lab}}(\tau^k) = \mathbf{1} \qquad (A.2)$$

holds. In the case of indistinguishable particles the Kraus operators are exactly the same, but we additionally must sum over all admissible k.

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